Bulletin 33:

TEMPERATURE AND TURBULANCE EFFECTS ON THE PARAMETER Δ IN THE STOCHASTIC MODEL FOR BOD AND DO IN STREAMS

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INTRODUCTION

The problem of stream pollution is of ever increasing concern today. It is a well known fact that use of the natural waterways of this country is rapidly becoming constrained due to the disposal of ever increasing quantities of man's wastes. If the recreational and aesthetic values of these natural resources are to be maintained, the dynamics of stream degradability and recovery must be accurately outlined and followed.

In order to set stream standards it is necessary to understand the factors which affect the state of pollution. The prediction of stream conditions, given a set of parameters and pollutional loads, is also necessary in setting limits on these loads. Once the effect of a pollution source upon the properties of a stream is understood, knowledgeable action may be taken to preserve the overall value of the reach.

The Problem

Many studies have been made to attempt a prediction of stream profiles given an initial set of conditions. The most formidable of these were conducted early by Streeter and Phelps (8), followed by Dobbins (4). Both groups of studies were of a deterministic nature, used to accurately predict the BOD and dissolved oxygen concentrations at a given point along a reach. A shortcoming of this type of analysis is that the one definitive concentration value derived via the formula and assumed to be exactly correct, may in fact be subject to considerable error. There is no consideration of the inherent variability of such values at a given point.

More recent studies by Thayer and Krutchkoff (9) consider the actual probability distribution of BOD and dissolved oxygen values at a given point along a reach. The uncertainty involved in the calculation of actual dissolved oxygen values at any given time was the major factor which prompted this work. This uncertainty results in the possibility of fish kills even if the dissolved oxygen levels calculated by the Dobbins theory were above the critical value. It was concluded that due to the statistical variability of actual dissolved oxygen values, the actual DO concentration could drop sufficiently below the calculated level so as to endanger aquatic life.

The work of Thayer and Krutchkoff (9) did not include a study of factors which would affect the principal parameter (Δ) used in determining the variances involved. This parameter was assumed to be a very small amount of pollution (BOD) or dissolved oxygen which was used by bacteria in a very short time interval.

The scope of this investigation is the study of temperature and turbulence effects upon the parameter Δ in an attempt to determine whether it is a mathematical constant which must be determined for each situation and cannot be generalized as suggested by Di Toro, Thomann, and O'Connor (3) or a physical parameter such as the rate constants (K_1, K_2, K_r, etc.) to which a general formula can be applied as suggested by Krutchkoff (5).

REVIEW OF LITERATURE

One of the most important considerations in characterizing a stream is that of its pollution assimilative capacity. This is the introduced pollutional load which produces a contaminant level equal to the minimum allowable established stream standard. The assimilative capacity will indicate tolerable influent levels that produce no excess degradation of the stream, if an increase in pollution is expected. This capacity of a stream to assimilate organic pollution was not predictable until the Streeter and Phelps (8) classical analysis. It was noted that the processes of deoxygenation and reaeration defined the oxygen sag curve of Figure 1. Initially, upon addition of organic pollution, there is a decrease in dissolved oxygen (deoxygenation) which occurs at a maximum rate and diminishes continuously toward zero. This initial decrease is due to bacterial action which uses dissolved oxygen in metabolism. The process of reaeration occurs simultaneously with that of deoxygenation and becomes the dominant factor when the food supply decreases and less oxygen is required. This is shown in the latter portion of the sag curve.

The curve of Figure 1 was defined in the work of Streeter and Phelps as

$$\frac{dD}{dt} = K_1 L - K_2 D$$

where L = BOD remaining at any time, t(ppm); K_1 = the BOD reaction rate constant (per day); D = dissolved oxygen deficit at any time (ppm); K_2 = the reaeration rate constant (per day); $K_1 L$ = decrease in oxygen due to biological processes; and $K_2 D$ = increase in oxygen due to physical reaeration. Integration of this equation yields:

$$D = \frac{K_1 L_A}{K_2 - K_1}\left(e^{-K_1 t} - e^{-K_2 T}\right) + D_A e^{-K_2 t}$$

where L_A = initial ultimate first stage BOD and D_A = initial oxygen deficit.

The latter equation was based on the assumption that pollution would be added at one point with no incremental addition in time along the reach. The only factors considered to affect the BOD and DO were nutrient and

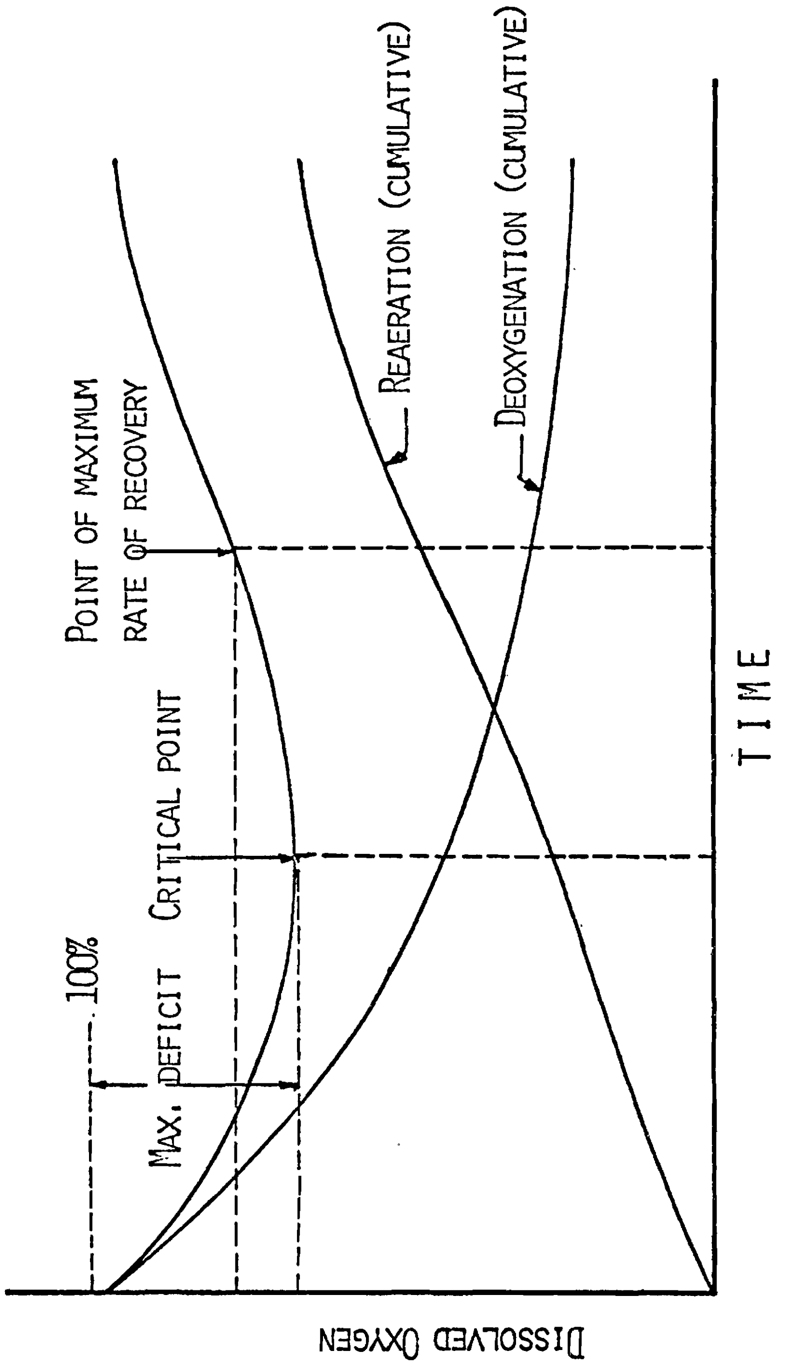

FIGURE 1. DEOXYGENATION, REAREATION, AND SAG CURVE

oxygen decrease due to bacterial metabolism and oxygen increase due to reaeration. Phelps also proposed equations for critical deficit and critical time as well as inflection deficit and time. They are as follows:

$$t_c = \frac{1}{K_1\ (f-1)} \log \left\{ f \left[1-(f-1)\ \frac{D_A}{L_A} \right] \right\}$$

$$f = \frac{K_2}{K_1}$$

$$D_c = \frac{L_A e^{-K_1 t_c}}{f}$$

$$t_i = \frac{\log f}{K_1 (f-1)} + t_c$$

$$D_i = \frac{f+1}{f^2} L_A e^{-K_1 t_i}$$

The inadequacy of the Streeter--Phelps analysis was recognized by H. A. Thomas (10) who introduced a rate constant, K_3, to account for the removal or addition of BOD by deposition or scour. If deposition occurs K_3 will be positive and if scour, or addition of BOD from bottom deposits, occurs K_3 will be negative. The K_3 rate constant may be used to include all changes in BOD excluding that of bacterial metabolic activity.

More recent work was published by W. E. Dobbins (4) in which he incorporated more factors and thus attempted a closer approximation of actual stream conditions. Though Dobbins' work was the best general theory to date, the variability of conditions was so great for different waterways, that his model at best was only a rough approximation. His equations were based on the following assumptions:

1. The stream flow is steady and uniform.

2. The process for the stretch as a whole is a steady state process, the conditions at every cross section being unchanged with time.

3. The removal of BOD by both bacterial oxidation and sedimentation or adsorption or both are first order reactions, the rates of removal at any point being proportional to the amount present.

4. The removal of oxygen by the benthal demand and by plant respiration, the addition of oxygen by photosynthesis, and the addition of BOD from the benthal layer or local runoff are all uniform along the stretch.

5. The BOD and oxygen are uniformly distributed over each cross section, thus permitting the equations to be written in the usual one-dimensional form.

The parameters which Dobbins employed were:

1. K_1 = removal rate of BOD and oxygen by bacterial action.

2. K_2 = surface reaeration rate.

3. K_3 = BOD removal rate due sedimentation or adsorption.

4. D_B = benthal demand for oxygen.

5. L_a = rate of pollution addition via runoff and tributaries.

6. D_L = Longitudinal disperson factor.

The longitudinal disperson factor was found to be negligible for streams of a non-tidal nature. Considering this, the following BOD and dissolved oxygen equations were developed:

$$L = L_A e^{-(K_1 + K_3)t} + \frac{L_a}{K_1 + K_3}\left[1 - e^{-(K_1 + K_3)t}\right]$$

and

$$D = \frac{K_1 \left\{L_A - \frac{L_a}{K_1 + K_3}\right\}\left\{e^{-(K_1 + K_3)t} - e^{-K_2 t}\right\}}{K_2 - (K_1 + K_3)} + D_A e^{-K_2 t}$$

$$+ \left\{\frac{D_B}{K_2} + \frac{K_1 L_a}{K_2(K_1 + K_3)}\right\}\left\{1 - e^{-K_2 t}\right\}$$

If there is no further external pollution addition (L_a), no settling or scour (K_3), and no oxygen demand from bottom organisms (D_B), the above equations resemble the Streeter--Phelps equations.

In an attempt to incorporate the addition of oxygen by photosynthesis, Camp (1) proposed the following equations:

$$L = L_A - \left[\frac{P}{(K_1 + K_3)}\right] e^{-(K_1 + K_3)t} + \frac{P}{(K_1 + K_3)}$$

and

$$D = \frac{K_1}{K_2 - K_1 - K_3}\left[L_A - \frac{P}{(K_1 + K_3)}\right]\left[e^{-(K_1 + K_3)t} - e^{-K_2 t}\right]$$

$$+ \frac{K_1}{K_2}\left[\frac{P}{K_1 + K_3} - \frac{\alpha}{K_1}\right]\left\{1 - e^{-K_2 t}\right\} + D_A e^{-K_2 t}$$

where p = rate of BOD addition from bottom deposits (ppm) and α = rate of production of dissolved oxygen by algae (ppm/day) as determined by the light-and-dark-bottle technique.

Since all the theories outlined yield, for any downstream point, one value for pollution concentration and one value for dissolved oxygen concentration, Thayer and Krutchkoff (9) developed a statistical model which considered the actual probability distribution of values. This would lead to a determination of the amount of time that the actual dissolved oxygen value was outside of the allowable limits. In setting up the model, the same parameters as mentioned in Dobbins' work, with the exception of the longitudinal dispersion factor, were assumed to be operating in the stream. In addition to Dobbins' five assumptions, one more was adopted. It was assumed that the changes in both pollution and dissolved oxygen occur in small units of size, (Δ). The problem was viewed as a stochastic birth and death process with BOD and dissolved oxygen being increased and decreased by small amounts (Δ) in very short time intervals (h). The parameter Δ can be thought of as a small "bundle" of pollution or dissolved oxygen which is not used by the bacteria until it reaches the size Δ and then it is used in the short interval (h). After this the bacteria must wait for another "bundle" to reach the size Δ. Thus, it was assumed that there is a series of very short feeding and non-feeding periods by the bacteria.

The term state was used to describe a BOD or dissolved oxygen condition. A change of size Δ constituted a change of one state. Thus there

were Δppm/state and any BOD or dissolved oxygen concentration could be expressed in terms of state by dividing the concentration by Δ as:

$$\text{state } M = L_m(\text{ppm}) \div (\Delta\text{ppm/state})$$

The probabilities for possible changes in state in time h were considered and the same equations which Dobbins arrived at deterministically were obtained statistically. Here, though, the variances could be found which would determine ranges of values at critical times. The importance of the Δ term lies in the fact that the variance of BOD and dissolved oxygen are directly proportional to it and are calculated with it.

The first step in determining Δ was a stream survey charting BOD and dissolved oxygen values along a given reach. From this, the stream parameters (K_1, K_2, K_r, L_A) were estimated and the BOD and dissolved oxygen sample variances, (S^2), were calculated by the sum of squares statistical method. Equations were developed by which variances (σ^2) could be calculated in terms of Δ for three different initial conditions. The estimate for Δ was then easily determined by:

$$\hat{\Delta} = \frac{S^2(t)}{\sigma^2(t)/\Delta}$$

An actual stream survey showed that the inherent error in the BOD test distorted the $\hat{\Delta}$ values as compared with that obtained from the dissolved oxygen data. Therefore, the parameter Δ was obtained by using a weighted average of dissolved oxygen values. From these results it was possible to calculate probabilities of having different dissolved oxygen concentrations for different initial conditions. In turn, initial conditions could be obtained based upon the probabilities. For example, a limit might be set on the probability of having a DO below 4 ppm. From this, the amount of BOD which could be added before the probability was reached would be calculated. In this way, the allowable amount of BOD added would be based on a probability of having a certain DO level.

To test the validity of this model, applications were made to the Sacramento River in California. This is a relatively clean river but the pollution influx is ever increasing with increased population and industrial use. A stream survey typically resulted in defining an oxygen sag curve which exhibited largest sample variances at the critical deficit. From the dissolved oxygen data the mean curve was plotted as well as the lower 10% and 20% confidence level variance curves (approximately 20% of the data points should lie below the 20% curve). This resulted in the curves of Figure 2 with the variance function changing with time as shown in Figure 3. The fact that the variance was greatest at the critical deficit is important since fish kills are more likely at this point. If an increase in pollution was allowed based upon consideration of the mean curve only, while the 10% lower confidence limit controlled, the resulting increased deficit would increase the possibility of fish kills.

With the information already obtained it was possible to predict new dissolved oxygen profiles as the river state changed. With these results it would be possible for governmental agencies to put realistic restrictions on allowable amounts of pollution.

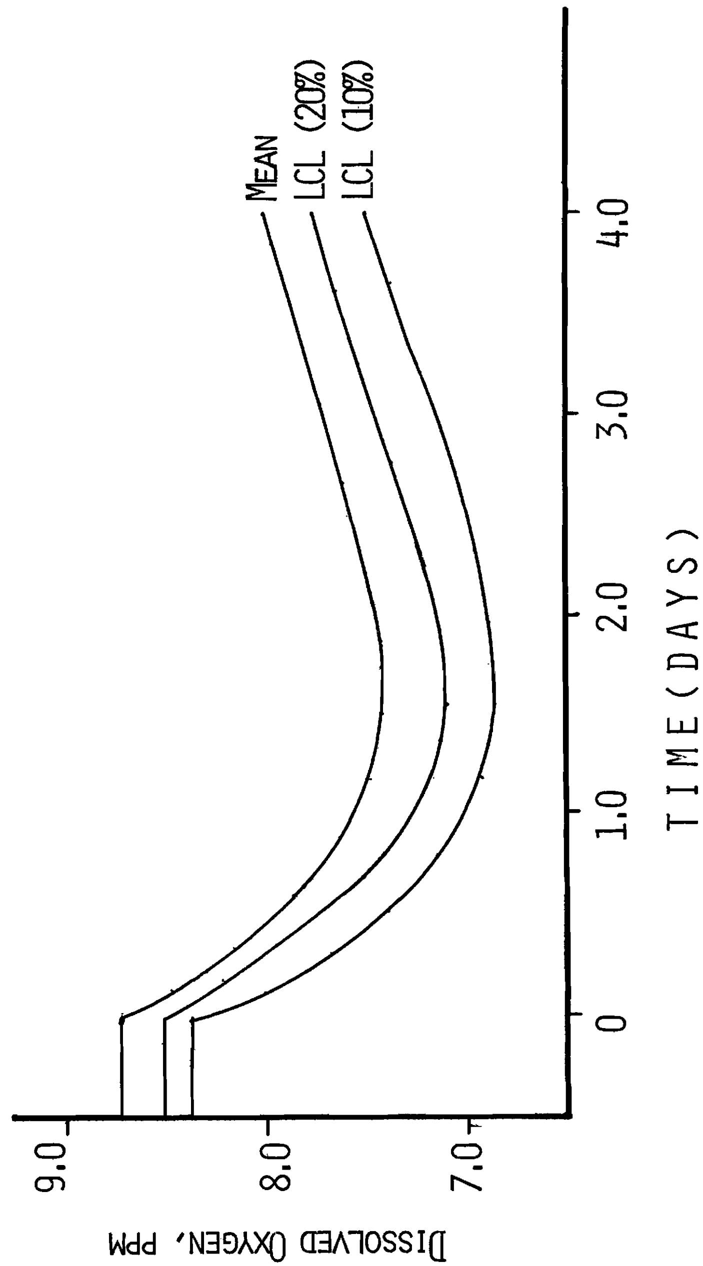

FIGURE 2. MEAN AND LOWER CONFIDENCE LIMITS

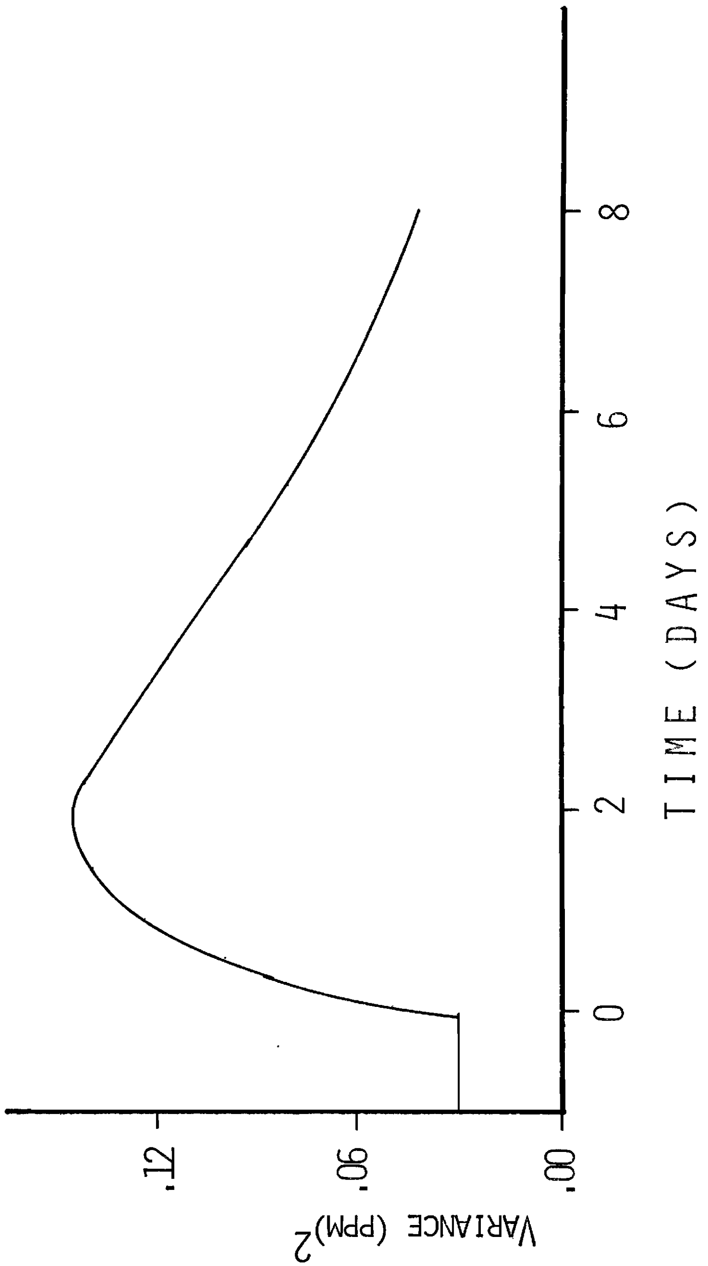

FIGURE 3. VARIANCE - TIME FUNCTION

EXPERIMENTAL PROCEDURE

The laboratory portion of this study was concerned with the effect which varying temperatures and turbulences would have upon the value of the parameter Δ. This necessitated development of a reliable method for determining the parameter Δ under various conditions. Laboratory tests performed by Thayer (9) to determine Δ were used as a basis for the experimental procedure in this study.

The most critical aspect of the laboratory work was development of a series of simulated dissolved oxygen sag curves. One curve was needed for each different set of initial conditions in order to obtain the parameters of the Dobbins equation. Thirteen test runs were required to obtain a general trend of the necessary parameters. Each run consisted of periodic dissolved oxygen measurements of five 10 liter samples to completion of the sag curve. These runs were distributed as follows:

1. Five different temperatures, constant turbulence rate.

2. Five different temperatures, a second constant turbulence rate.

3. Five different turbulence rates, constant temperature.

Two of these runs overlap resulting in a net of thirteen separate observations.

The runs were carried out in ten liter plastic waste cans. Each can represented a parcel of river water to which pollution (nutrients) was added. The individual runs consisted of five identical cans, each filled with ten liters of water from the Virginia Polytechnic Institute's duck pond. This water contained an abundence of fresh water and sewage bacteria with no chlorine to hinder the growth of bacteria.

Simulation of pollution influx to a stream was accomplished by addition of a growth substrate to the water in the test cans. The simulated pollution substrate consisted of 40 mg/l dextrose, 1 part nitrogen to 40 parts dextrose, and 1 part potassium to 200 parts dextrose. Nitrogen was added as ammonium chloride and potassium in the form of potassium phosphate.

The report of Thayer and Krutchkoff (9) indicated a long lag (24 hours) between substrate addition and appreciable initial oxygen depletion in the water of the test cans. To shorten this initial lag period, addition of a bacterial enrichment solution was used to "seed" the test solution. This "seed" solution reduced the time required to simulate an oxygen sag curve by introducing dense concentrations of bacterial cells acclimated to the conditions of the test. Trial additions of various enrichment solutions indicated that the most applicable "seed" solution was composed as follows:

875 ml	pond water
25 ml	settled sewage
30 mg/ℓ	glucose
20 mg/ℓ	yeast extract
120 mg/ℓ	peptone
10 ml/ℓ	phosphate buffer

Using the "seed" solution the lag decreased from 24 hours to below 12 hours. In preparing the "seed" solution, the glucose, yeast extract, peptone, and phosphate buffer solution were dissolved in 90 ml distilled water and diluted to 1 liter using the pond water containing bacteria. This solution was aerated for 24 hours prior to use.

Prior to each run, five cans were filled with 10 liters of pond water each and placed in a constant temperature room at 18°C. The samples were paddle stirred using electric variable speed motors with rotation speeds determined by use of a strobe light. Each sample was stirred for 24 hours to insure a constant temperature and homogeneous conditions. During this period the seed was allowed to aerate in a separate container. The dextrose feed and 20 ml of "seed" solution were added to each test solution upon completion of the 24 hour stirring and aeration period. An initial dissolved oxygen reading was taken using a Beckman Probe which was calibrated prior to each run via the Winkler Method as outlined in Standard Methods (7). The stirring was continued and dissolved oxygen readings were taken periodically to obtain the sag curve simulations.

Other parameters (K_1, K_2, K_r, L_A) required to calculate Δ were evaluated in the system with the benthal demand assumed to be zero. The

Thomas Graphical Method (10) was used to evaluate the parameters K_1 and L_A. The procedure as outlined in Standard Methods (7) for the determination of BOD was employed with evaluations of BOD made initially and at periodic 12 hour intervals. The river deoxygeneration constant, K_r, was also determined through the use of BOD data via the BOD equation:

$$K_r = \frac{1}{t} \ln \frac{L_A}{L_t}$$

A value for the reaeration constant, K_2, was determined by a trial and error solution of the Dobbins equation after all other values were established.

Fish aquarium heaters were used to maintain the test solutions at the desired temperatures which ranged from 18°C to 38°C with intervals of 5°C. Turbulence variations were induced through use of 1 square inch flat paddles, 3 square inch flat paddles, and varying stirring speeds. The turbulence values are reported as the flocculation parameter, G:

$$G = \sqrt{\frac{C_D A\ v^3}{2\ V\ \nu}}$$

where C_D = drag coefficient (1.8 for flat plate); A = paddle area; v = velocity of paddle relative to water; V = tank volume; ν = kinematic viscosity.

EXPERIMENTAL RESULTS

At the beginning of each run, a series of BOD tests was set up to obtain data for use in the Thomas Graphical Method for evaluating K_l and L_A. The results of this data for the 18°C and G = 38.1 sec $^{-1}$ run are reported in Table 1. Also reported in this table are the average values for BOD which are required to calculate $(t/y)^{1/3}$, as used in the Thomas Graphical Method.

The BOD data of Table 1 was plotted and the resulting curve is shown in Figure 4. The lag in the BOD curve dictated a t = 0 12 hours into the run. The values of $(t/y)^{1/3}$ were calculated using the adjusted t values. This resulted in the curve shown in Figure 4. Using the slope, A, of this curve as well as its intercept, B, the parameters were calculated as:

$$K_1 = \frac{6B}{A} = 0.08 \text{ per day}$$

$$L_A = \frac{1}{K_1 A^3} = 58 \text{ ppm}$$

This method was used to determine the K_1 and L_A values in each of the nine runs. These values are listed in Table 3.

The river deoxygenation constant, K_r, was determined by use of the following equation:

$$K_r = \frac{1}{t} \ln (L_A/L_t)$$

The BOD data for the 18°C and G = 38.1 sec^{-1} run is shown in Table 2. This method was repeated in each run. The results are shown in Table 3.

Table 1. Thomas Graphical Data

Sample	time (days)	BOD (mg/ℓ)	$BOD_{ave.}$ = Y (mg/ℓ)
1		0.00	
2		0.10	
3	0.5	0.07	....
4		0.00	
5		0.03	
1		2.23	
2		3.12	
3	1.0	2.25	2.26
4		1.84	
5		1.92	
1		4.44	
2		4.60	
3	1.5	4.20	4.40
4		4.07	
5		4.22	
1		6.70	
2		6.82	
3	2.0	6.30	6.42
4		5.91	
5		6.20	
1		8.47	
2		8.80	
3	2.5	8.20	8.35
4		7.97	
5		8.20	

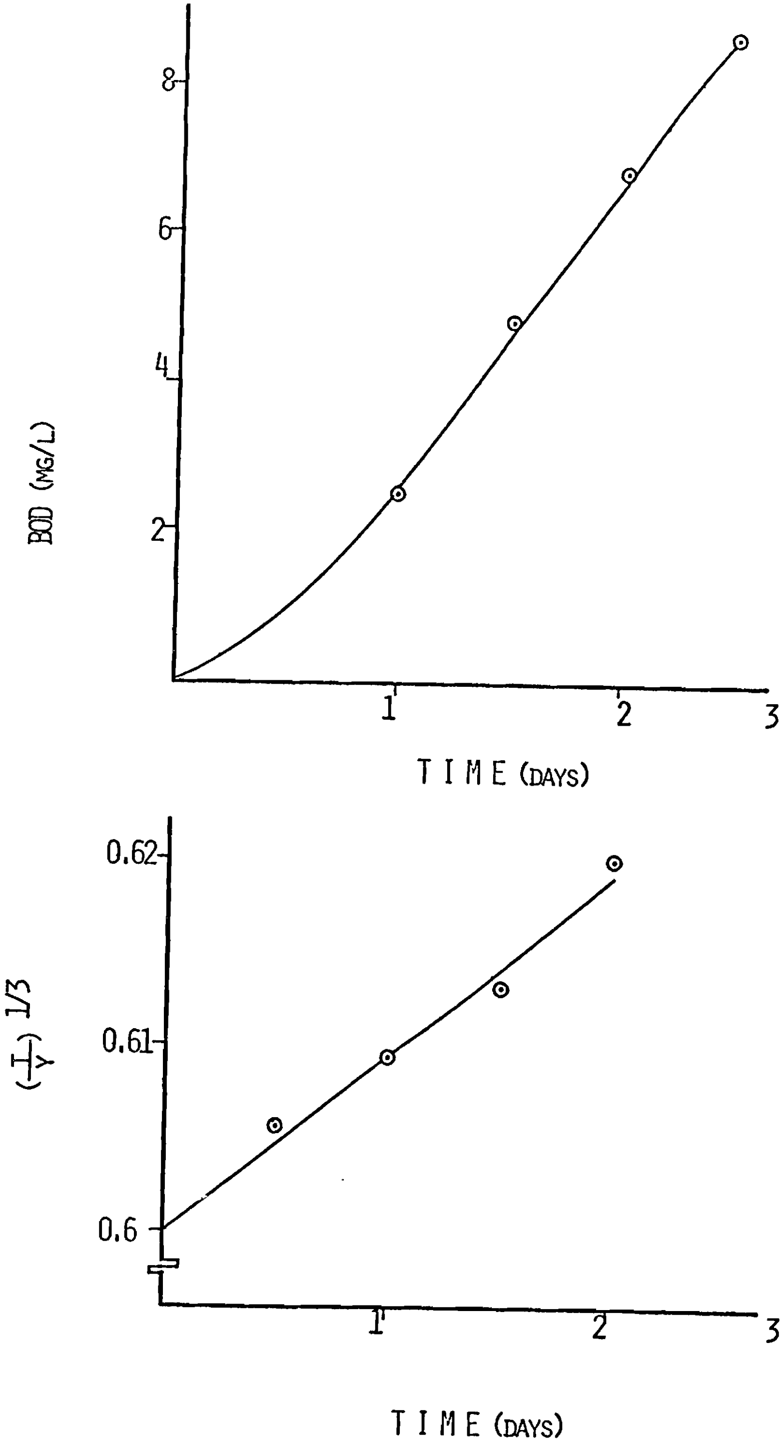

FIGURE 4. THOMAS GRAPHICAL CURVES

Table 2. BOD and K_r Data

Sample	L_A (ppm)	L_2 day (ppm)	K_r per day	avg K_r
1	56.7	35.0	0.24	
2	55.1	32.8	0.26	
3	61.8	33.9	0.30	0.25
4	59.4	35.9	0.25	
5	56.6	36.5	0.22	

Dissolved oxygen readings were taken periodically throughout each run and curves were plotted from this data. These curves are shown in Figure 5 through Figure 9. The changes in the sag curve with changes in temperature and turbulence are illustrated in these figures. Also, the initial oxygen deficit, D_A, the critical oxygen deficit, D_c, and the critical time, t_c, were read from each graph and tabulated in Table 3.

The five duplicate samples yielded five DO values at any point in time on the oxygen sag curve. These five values were employed to calculate the sample variances (S^2) for each individual time by:

$$S_c^2 = \frac{1}{4}\left(\Sigma c_i^2 - 5\bar{c}^2\right)$$

where c = dissolved oxygen concentration (ppm). For every value of S_c^2 a value for σ_c^2/Δ value was calculated by:

$$\sigma_c^2/\Delta = D_A \left\{1 - e^{-K_2 t}\right\} e^{-K_2 t} \quad +$$

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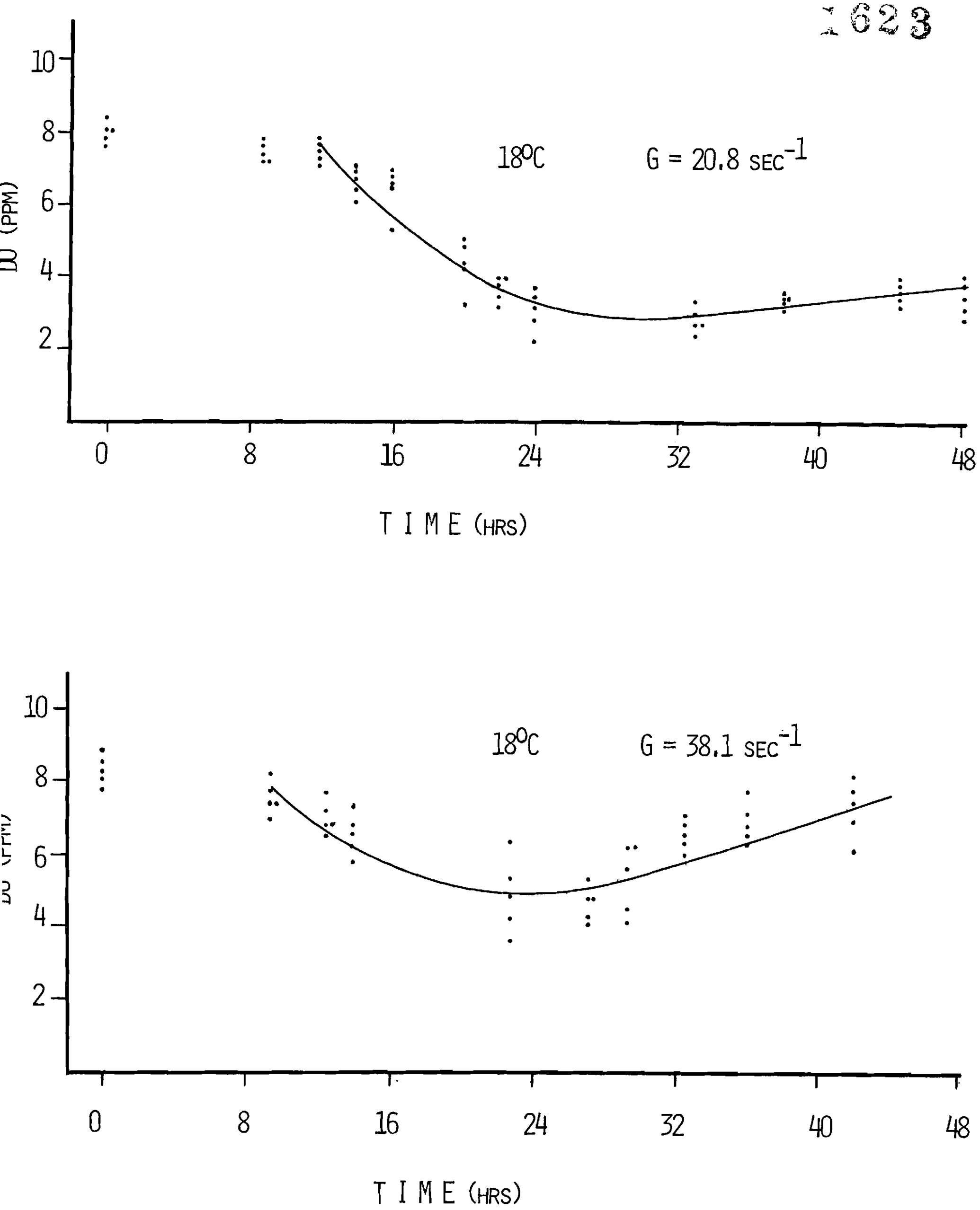

Figure 5. Dissolved Oxygen Curves

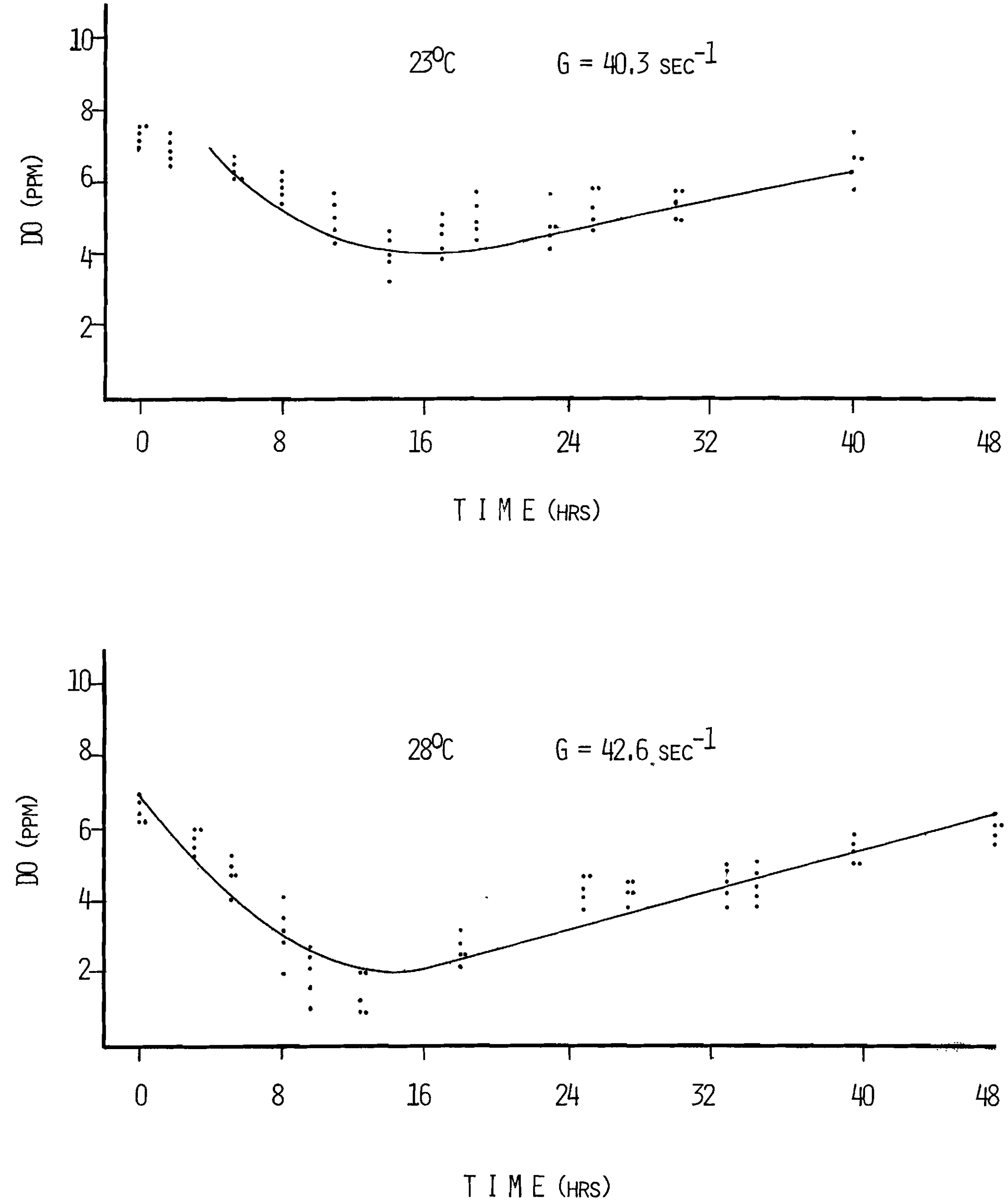

Figure 6. Dissolved Oxygen Curves

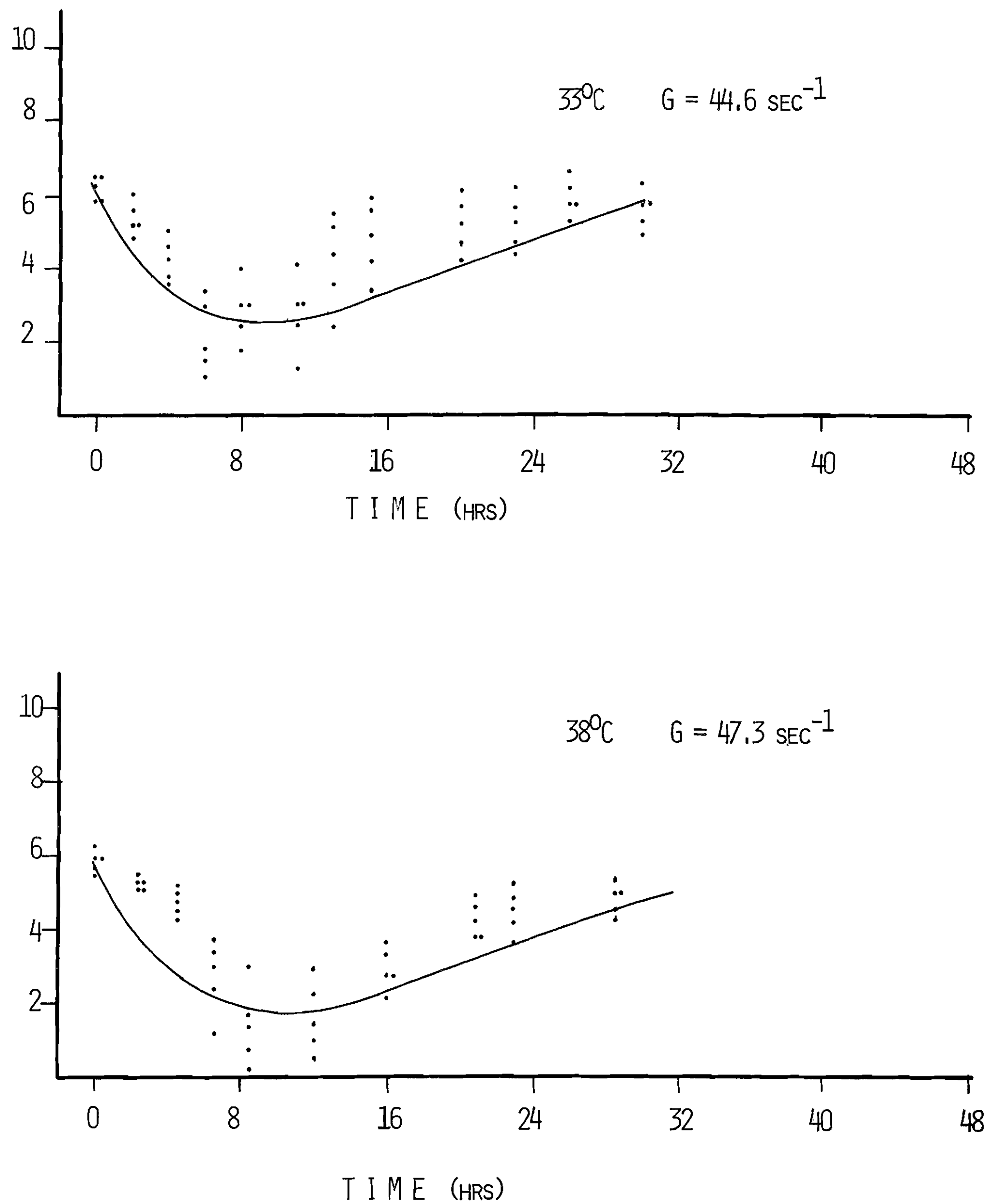

Figure 7. Dissolved Oxygen Curves

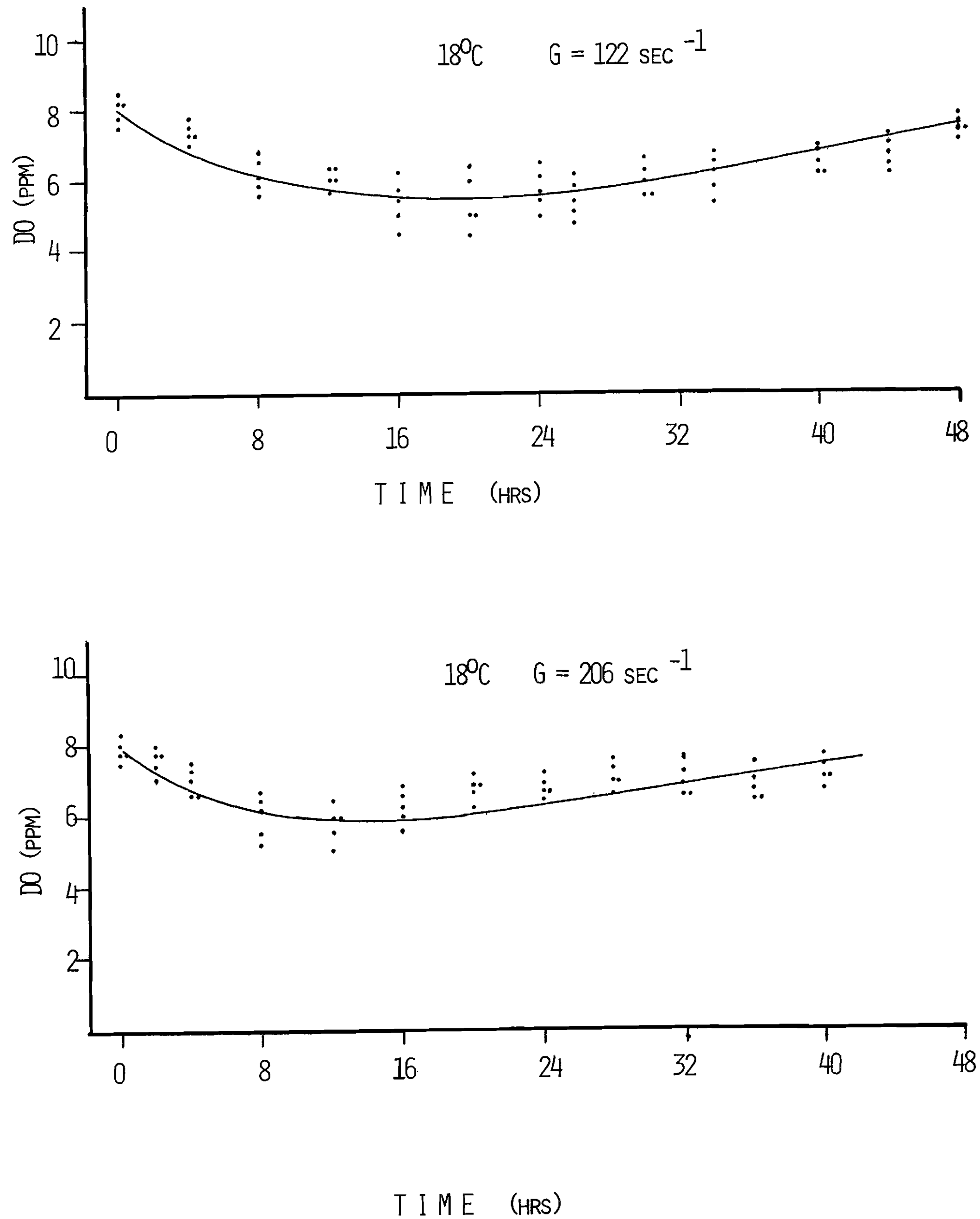

Figure 8. Dissolved Oxygen Curves

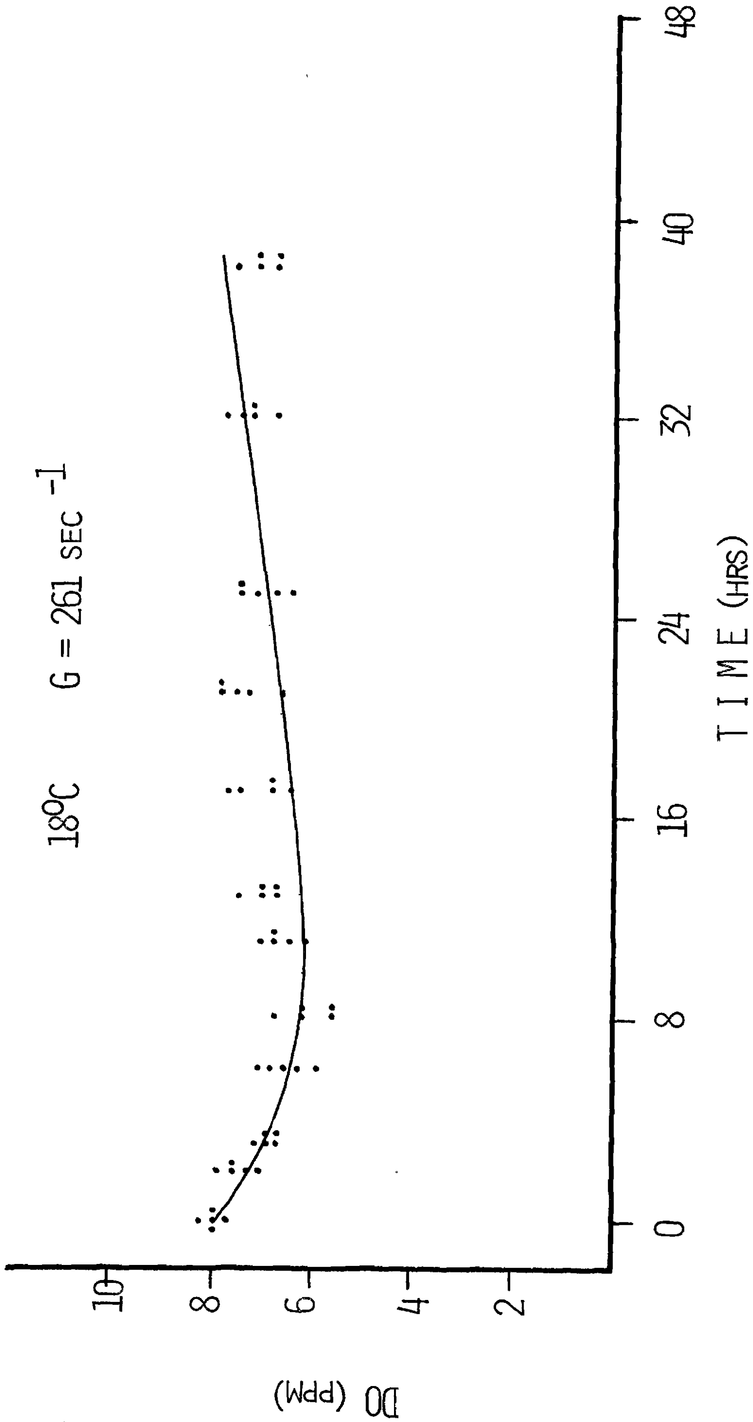

FIGURE 9. DISSOLVED OXYGEN CURVE

$$L_A \left\{ \frac{K_1}{K_2 - K_r} \right\} \left\{ e^{-K_r t} - e^{-K_2 t} \right\} \left[1 - \left\{ \frac{K_1}{K_2 - K_r} \right\} \left\{ e^{-K_r t} - e^{-K_2 t} \right\} \right]$$

With this data a value of $\hat{\Delta}$ was calculated for each value of σ_c^2/Δ by use of the following equation:

$$\hat{\Delta} = \frac{S_c^2}{\sigma_c^2/\Delta}$$

The values of $\hat{\Delta}$ were averaged for each run and reported in Table 3 as Δ. The variances of Δ are listed in the same table as S_Δ^2

Graphs of the relationship between Δ and temperature as well as Δ and G are shown in Figure 10.

The regression method was used to obtain one formula for the relationships of Figure 10. The formula obtained was:

$$\Delta = .08 + .007T - .00061G$$

Table 3. Temperature - Parameter Relationships

Temp (°C)	G (sec^{-1})	K_1 (/day)	K_2 (/day)	K_r (/day)	L_A (ppm)	D_A (ppm)	D_c (ppm)	t_c (HRS)	Δ	S^2_Δ
18	261	0.080	2.04	0.182	58	1.2	2.8	10	0.045	0.0018
18	206	0.085	1.85	0.210	52	1.2	3.2	16	0.086	0.0078
18	122	0.082	1.67	0.228	54	1.1	3.7	20	0.135	0.005
18	38.1	0.080	1.50	0.250	58	1.0	4.2	24	0.180	0.023
23	40.3	0.102	1.62	0.270	55	1.2	4.2	16	0.196	0.0035
28	42.6	0.127	1.65	0.304	60	1.0	6.2	14	0.259	0.0084
33	44.6	0.159	1.77	0.342	56	0.9	4.1	10	0.294	0.0409
38	47.3	0.20	1.85	0.385	54	0.4	5.0	10	0.302	0.006

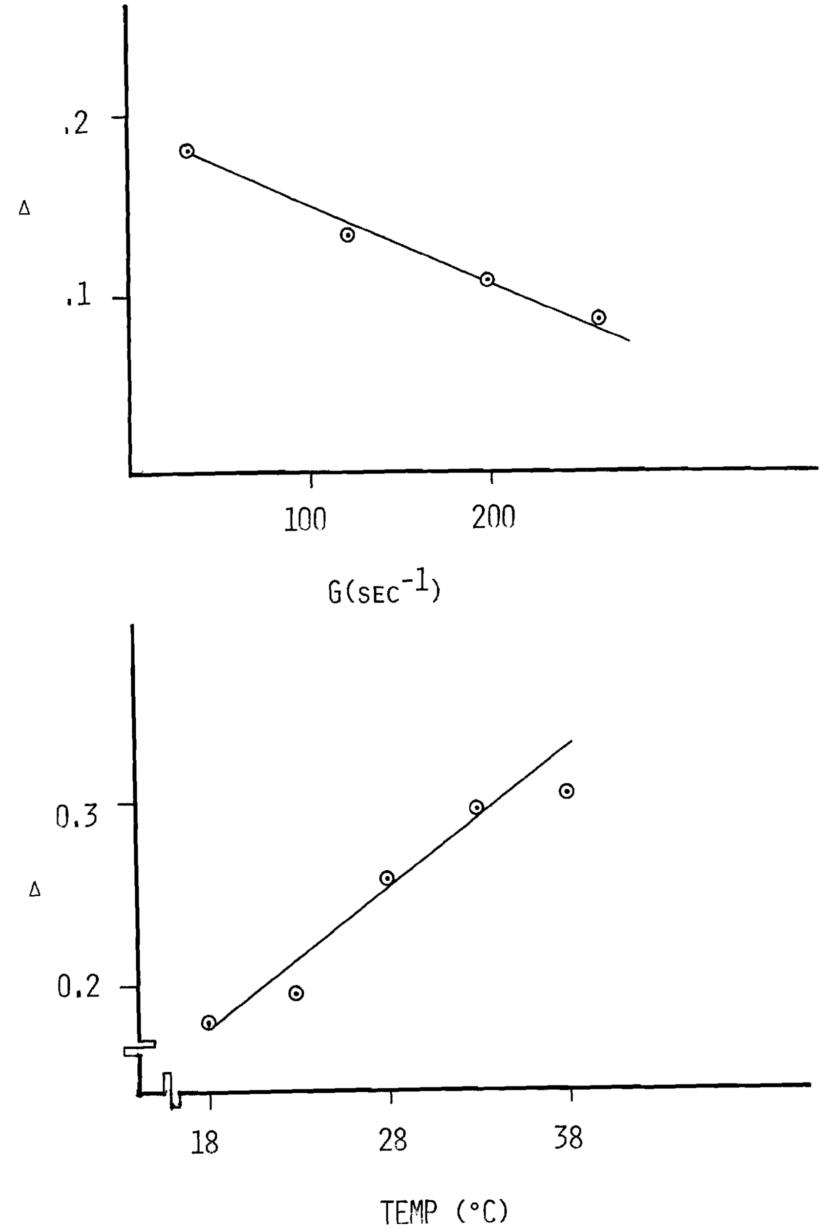

FIGURE 10. Δ, TEMPERATURE, AND G RELATIONSHIPS

DISCUSSION OF RESULTS

The oxygen sag curves reported in Figures 5 through 9 follows the Dobbins equation with the exception of the 18°C, G = 20.8 sec^{-1} run. This curve dropped to a critical deficit of 6 ppm below saturation and leveled off. This could be a result of incomplete mixing.

The behavior of the rate constants with changing conditions followed the patterns which were anticipated. The reaeration constant, K_2, increased with increasing temperature and turbulence and K_1 increased with increasing temperature as did K_r.

A study of the individual oxygen sag curves revealed that, as Thayer and Krutchkoff (9) predicted, the variability of concentrations was greatest at the critical times and followed much the same behavior as shown in Figure 3. This is illustrated in the graphs of Figure 6 which indicate the greatest variances exist between the five sample values conforming to the critical deficit.

The parameter Δ was calculated in each case using dissolved oxygen data rather than BOD data since as Thayer and Krutchkoff (9) noted, there is an error inherent to the BOD test method. Therefore, a weighted average of dissolved oxygen data was used to calculate Δ. This reduced the possibility of error.

The graphs of Figure 10 relating Δ to temperature and the flocculation parameter, G, show that a clear physical relationship exists between these variables. This relationship was formulated as:

$$\Delta = .08 + .007T - .00061G$$

This is a physical as opposed to a mathematical parameter since it is directly related to physical parameters, (T, G), and may be calculated via these parameters.

In the initial work of Thayer and Krutchkoff (9), the parameter Δ was necessary for prediction of dissolved oxygen concentrations for a given point in time. The formulation of Δ and the fact that it is a physical parameter enables a calculation of Δ given an initial temperature and flocculation parameter for a given nutrient condition.

The fact that nutrient conditions vary from stream to stream requires a determination of the relationship between T, G and Δ for each stream separately. The equation given here represents the relationship for the given nutrient conditions.

Krutchkoff (5) predicted "clearly Δ will be affected by such things as turbulence, temperature, type of BOD etc. For example, greater mixing will decrease Δ." Figure 10 illustrates the accuracy of this prediction. The value of Δ clearly decreases with mixing (G). The reason for the increase of Δ with temperature is not quite so obvious. That Δ does increase with temperature is, however, clearly seen in Figure 10.

CONCLUSIONS

1. The dissolved oxygen variance function as proposed by Thayer and Krutchkoff (7) was verified with largest variances realized at the critical dissolved oxygen deficits.

2. The parameter Δ is a physical rather than a mathematical factor.

3. The parameter Δ increases with increasing temperature and decreases with increasing turbulence by the equation:

$$\Delta = .08 + .007T - .00061G$$

for the nutrient conditions used here.

SUMMARY

With the realization that the natural environmental capacities of waterways were being altered through introduction of pollution, men such as Streeter, Phelps, Camp, and Dobbins devised methods to calculate the effects of pollution upon the assimilative capacity of a stream. The equations obtained through their work made possible calculations of BOD and dissolved oxygen concentrations if initial conditions and dynamic parameters were known.

A single specific estimated value for BOD and dissolved oxygen for a specific point in time is calculated via the Dobbins equation. Thayer and Krutchkoff (9) developed a statistical model to include the variability of BOD or dissolved oxygen values at any given point in time. This yielded a range of values over which the actual concentration might lie. The variance was found to be largest at the critical, or lowest, dissolved oxygen concentration.

The major assumption of Thayer and Krutchkoff (9) was that BOD and dissolved oxygen concentrations change by small amounts, Δ, in small time intervals, h. Knowledge of the value of Δ enabled calculation of the probability of having any specific dissolved oxygen concentration for any point in time.

The work of Thayer and Krutchkoff did not include formulation of Δ as a physical parameter which could be generalized in terms of other variables.

This study illustrates the effects of temperature and turbulence upon the parameter Δ. A series of simulated DO sag curves under varying conditions of temperature and turbulence were developed in the laboratory. These were accomplished by probe measurements of dissolved oxygen in a series of test solutions to which glucose feed solutions were added. The stream parameters (K_1, K_2, K_r, L_A,) were estimated and Δ was calculated under each test condition. A plot of temperature versus Δ and turbulence versus Δ demonstrated that the parameter was of a physical nature and could be formulated as follows:

$$\Delta = .08 + .007T - .00061G$$

This allows calculation of a generalized Δ value for specific environmental conditions obviating the need for downstream DO measurements.

This concept of Δ allows an investigator to predict a range of dissolved oxygen values given a set of initial conditions. The probability of any actual DO value falling within this range may also be obtained, allowing intelligent pollution control decisions to be made based on the probability function.

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